Jet Impingement on a Curved Surface

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Theme

FLOW near the impingement region of a two-dimensional incompressible jet over a curved surface is analyzed to predict curvature effects. The flow regime in the neighborhood of the stagnation point is divided into inviscid and viscous flow regions in a curvilinear coordinate system. The inviscid flow solution is obtained by solving full Euler's equations. The viscous flow is analyzed by using zeroth- and first-order boundary-layer equations, the latter accounting for the curvature effects. The numerical solutions are valid in the impingement region extending approximately to half the free jet width on either side of the stagnation point.

Contents

Formulation and Analysis

We choose a coordinate system comprising curves parallel to the surface and their normals (Fig. 1). The coordinate x is nondimensional with respect to the half-width r_b^* of the jet, and y and k (surface curvature) with respect to y_∞^*. The velocity components are nondimensional with respect to the fully developed jet velocity V_j^* at $(0,y_\infty^*)$ along the axis. The velocity profile of the jet is assumed to be of the form

$$V_j = -(1-x^{3/2})^2$$

The inviscid flow regime is governed by the following Euler's equation

$$\frac{\partial U}{\partial x} + \alpha \frac{\partial}{\partial y}\left\{(1+ky)V\right\} = 0$$

and

$$\frac{\partial w}{\partial x} + \alpha(1+ky)V\frac{\partial w}{\partial y} = 0$$

where $\alpha = r_b^*/y_\infty^*$ and the magnitude of α defines the geometry of the problem more specifically; w is the vorticity. The boundary conditions are

$$V(x,1) = V_j$$

and

$$U(x,1) = V(x,0) = 0$$

The zeroth-order viscous flow solution is obtained by Prandtl's boundary-layer equations

$$\frac{\partial u_0}{\partial x} + \alpha\frac{\partial v_0}{\partial \bar{y}} = 0$$

$$u_0\frac{\partial u_0}{\partial x} + \alpha v_0\frac{\partial u_0}{\partial \bar{y}} = U_0\frac{\mathrm{d}U_0}{\mathrm{d}x} + \alpha\frac{\partial^2 u_0}{\partial \bar{y}^2}$$

where U_0 is the inviscid flow distribution on the surface. The boundary conditions are

$$u_0(x,0) = v_0(x,0) = 0$$

and

$$u_0(x,\infty) = U_0$$

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The first-order boundary-layer equations are obtained from the following expressions (Refs. 1 and 2)

$$\frac{\partial u_1}{\partial x} + \alpha\frac{\partial}{\partial \bar{y}}(v_1 + k\bar{y}v_0) = 0$$

$$u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial \bar{y}} + \alpha\left(v_0\frac{\partial u_1}{\partial \bar{y}} + v\frac{\partial u_0}{\partial \bar{y}}\right)$$

$$= -\frac{\partial p_1}{\partial x} + \alpha\left[\frac{\partial^2 u_1}{\partial \bar{y}^2} + k\left\{\bar{y}\left(\frac{\partial^2 u_0}{\partial \bar{y}^2} - v_0\frac{\partial u_0}{\partial \bar{y}}\right) - u_0 v_0\right\}\right]$$

$$\frac{\partial p_1}{\partial x} = \frac{\partial}{\partial x}k\int_0^y u_0^2\mathrm{d}y + k\int_0^\infty (U_0^2 - u_0^2)\mathrm{d}y - w_0 v_1(x,0)$$

where $\bar{w}_0$ is vorticity on the surface. The boundary conditions are

$$v_1(x,0) = \left\{v_0 + \left(\frac{\mathrm{d}u_0}{\mathrm{d}x}\right)\bar{y}\right\}_{\bar{y}\to\infty} \qquad u_1(x,0) = v_1(x,0) = 0$$

and

$$u_1(x,\infty) = (\bar{y}w_0 - k\bar{y}U_0)_{\bar{y}\to\infty}$$

The inviscid flow velocity component V is assumed to have the following form

$$V = \sum_1^5 (-1)^n x^{3(n-1)/2} F_n$$

where F_n is a function of y alone. The zeroth- and first-order boundary-layer velocity components are assumed to be of the form

$$u_0 = \alpha\sum_{n=1}^5 a_n x^{(3n-1)/2} f_n'$$

and

$$u_1 = \alpha\sum_{n=1}^5 a_n x^{(3n-1)/2} l_n'$$

where f_n and l_n are functions of η only, $(\eta = \bar{y}/\sqrt{a_1})$ and the prime denotes differentiation. The values of the constant a_n are obtained from the inviscid flow solution. The above series of expressions, when substituted into the governing equations, give rise to a system of ordinary differential equations which are solved numerically using standard techniques on a CDC computer.

Discussion of Results

The inviscid flow velocity U is calculated in the entire flow regime, and in Fig. 2 the distribution of u on the surface for $k=0.2$ is presented. The velocity distribution U_0, with a maximum gradient at the stagnation point, increases with x, attains a gradual maximum at about $x=0.8$, and remains constant thereafter. Reduction in surface curvature results in a reduction of the magnitude of the inviscid flow velocity. It would be interesting to note that if the velocity U_0 is normalized with respect to maximum velocity $U_{\max}$ obtained at $x=1.0$, the distribution of $U_0/U_{\max}$ remains practically the same for all values of surface curvature considered. This is a direct consequence of the following relations that are found to hold good for all values of k

$$F_2'(0) \doteq 2F_1'(0)$$

and

$$F_1'(0) \doteq F_3'(0)$$

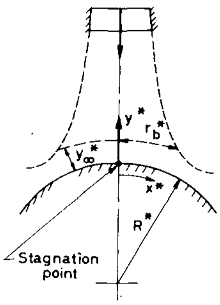

Fig. 1 Schematic diagram of the problem.

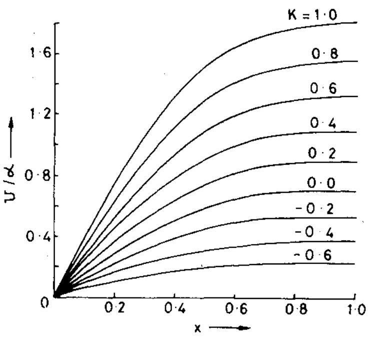

Fig. 2 Inviscid flow velocity distribution on the surface.

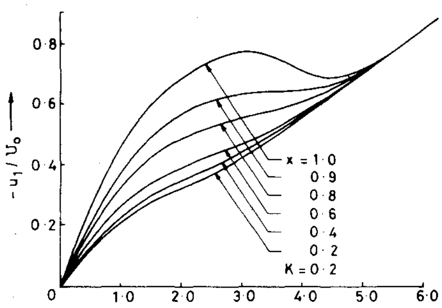

Fig. 3 Distribution of first-order boundary-layer velocity.

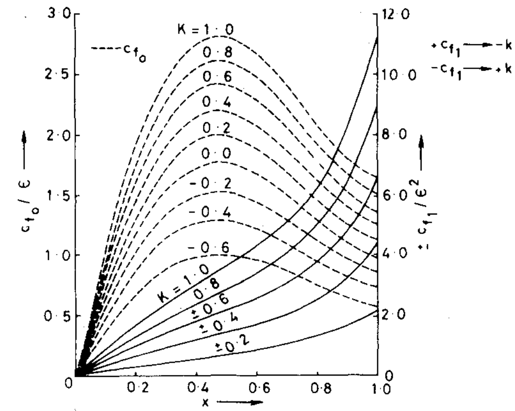

Fig. 4 Skin-friction distribution.

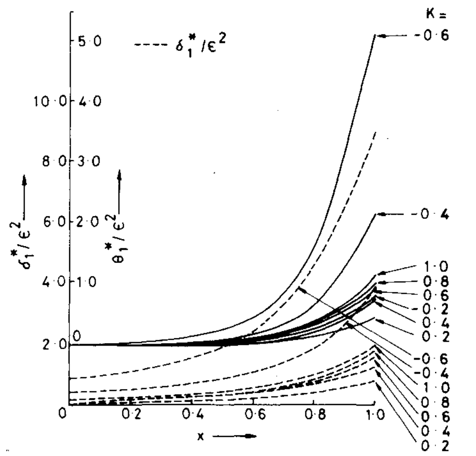

Fig. 5 First-order displacement and momentum thicknesses.

Consequently U_0 can be written as

$$U_0 = \alpha(x - 0.8x^{2.5} + 0.25x^4)F_1'(0)$$

The zeroth-order boundary-layer velocity distribution u_0/U_0 is independent of k and exhibits the well known typical features of laminar boundary layers.

One of the principal effects of longitudinal surface curvature is the variation of first-order boundary-layer velocity profiles with surface curvature. The distribution of u_1/U_0 for $k = 0.2$ is shown in Fig. 3. Each of these profiles exhibits a point of inflexion and matches linearly with the outer boundary condition. With increase in x the point of inflexion of the velocity profile moves up and becomes more and more pronounced. The velocity profiles for all curvatures considered exhibit these features, with a change of sign for concave surfaces.

The skin-friction coefficient is given by the expression

$$c_f = 2\tau_w^*/\rho V_j^2 = \epsilon\, c_{f_0} + \epsilon^2\, c_{f_1}$$

where τ_w^* is the wall shear stress and ϵ is the reciprocal of the square root of the Reynolds number.

The zeroth-order skin-friction coefficient c_{f_0} (Fig. 4) attains a maximum value at about $x = 0.5$ and thereafter exhibits a drooping characteristic in accordance with the distribution of outer inviscid flow. Reduction in curvature results in a reduction in skin friction due to reduction in U_0 and its gradients. The first-order skin friction due to surface curvature (Fig. 4) increases rapidly as x approaches unity. An increase in curvature results in an increase in first-order skin friction, which means that total skin friction is much lower in the case of a convex surface and higher in the case of a concave surface than what is obtained from Prandtl's equation.

The displacement thickness δ^* and momentum thickness θ are defined as

$$\delta^* = \int_0^y \left(1 - \frac{u}{U_0}\right) dy = \epsilon\delta_0^* + \epsilon^2\delta_1^*$$

and

$$\theta = \int_0^y \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy = \epsilon\, \theta_0 + \epsilon^2\theta_1$$

The distribution of zeroth-order thicknesses δ_0^* and θ_0 are just the same as obtained by Prandtl's boundary-layer equations. Effect of curvature on first-order displacement and momentum thicknesses can be readily seen from Fig. 5. The value of these is zero for zero curvature. With increase in x values of δ_1^* and θ_1 increase, and increase more rapidly with increase in the value of curvature for concave surfaces. This follows from the fact that reduction in the magnitude of first-order velocities is greater and faster for concave surfaces than for convex surfaces for the same amount of incremental change in the value of curvature.

It is thus found that the velocity profiles of the present nonsimilar flow solution bear qualitative resemblance to those obtained by similarity solutions[2] and by direct methods.[3] The skin friction is found to decrease with reduction in curvature and the contribution of first-order skin friction is positive for concave surfaces and negative for convex surfaces.

References

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[3]Bluston, H.S. and Paulson, R.W., "Non-Similar Second Order Boundary Layer Solutions for Subsonic Flow Over Curved Surfaces, *Journal de Mecanique*, Vol. 10, June 1971, pp. 163-189.